

## Intro to Logarithms Unit

Sound intensity  $10^x$   
Decibels  $10x + 120$

1 Find the decibel ratings of these sounds.

a. A passing subway train with sound intensity reading of  $10^{-0.5} \frac{\text{watts}}{\text{m}^2}$

$$10(-.5) + 120 = 115 \text{ decibels}$$

b. An excited crowd at a basketball game with sound intensity reading of  $10^{1.25} \frac{\text{watts}}{\text{m}^2}$

$$10(1.25) + 120 = 132.5 \text{ decibels}$$

2 Find these common (base 10) logarithms without using a calculator and explain your reasoning.

a.  $\log 100,000$

$$\text{Since } 10^5 = 100,000$$

$$\log 100,000 = 5$$

b.  $\log 0.001$

$$\text{Since } 10^{-3} = .001$$

$$\log .001 = -3$$

c.  $\log (10^{4.75})$

4.75 because

log finds exponent  
on base 10

$$10^{4.75} = 10^{4.75}$$

d.  $\log 1$

$$\text{Since } 10^0 = 1$$

$$\log 1 = 0$$

3 Find the decibel ratings of these sounds.

a. A door slamming with sound intensity  $89 \frac{\text{watts}}{\text{m}^2}$

$$10^x = 89$$

$$\log 89 = x$$

$$1.9 = x$$

$$10(1.9) + 120 = 139 \text{ decibels}$$

b. A radio playing with sound intensity  $0.005 \frac{\text{watts}}{\text{m}^2}$

$$10^x = .005$$

$$\log .005 = x$$

$$-2.3 = x$$

$$10(-2.3) + 120 = 97$$

4 Pure water has a pH of 7. Liquids with pH less than 7 are called acidic; those with pH greater than 7 are called alkaline. Typical seawater has pH about 8.5, soft drinks have pH about 3.1, and stomach gastric juices have pH about 1.7.

a. Which of the three liquids are acidic and which are alkaline?

Acidic: soft drinks + gastric juices

alkaline: seawater

b. Find the concentration of hydrogen ions in seawater, soft drinks, and gastric juices. From p. 380 #8,  $\text{pH} = -\log H^+$ , so  $-\text{pH} = \log H^+$

$$\text{seawater: } 10^{-8.5} \approx .000000003$$

$$\text{soft drinks: } 10^{-3.1} \approx .0008$$

$$\text{gastric juices: } 10^{-1.7} \approx .02$$

c. Explain why it is correct to say that the concentration of hydrogen ions in gastric juices is about 25 times that of soft drinks.

$$\text{Since } .0008 \cdot 25 = .02$$

$$\text{or } \frac{.02}{.0008} = 25$$

d. If a new soft drink has a hydrogen ion concentration that is one-fifth that of typical soft drinks, what is its pH?

$$\frac{1}{5} (.0008) = .00016$$

$$\text{pH} = -\log .00016 = 3.8$$

5 Use algebraic reasoning with logarithms to solve the following equations for  $x$ .

a.  $\log x = 2$

$$10^2 = x$$

$$100 = x$$

b.  $15 = 10^x$

$$\log 15 = x$$

$$1.176 = x$$

c.  $\frac{5(10)^{2x}}{5} = \frac{60}{5}$

$$10^{2x} = 12$$

$$\frac{\log 12}{2} = \frac{2x}{2}$$

$$.5396 = x$$

d.  $10^{3x-1} = 100,000$

$$\log 100000 = 3x - 1$$

$$5 = 3x - 1$$

$$6 = 3x$$

$$2 = x$$

For #6-9 answers, please see your teacher.

- 14 Solve the following exponential equations and then explain how the strategies used are similar to what you use in solving linear equations.

a.  $10^{x+2} = 100,000$

$$\log 100000 = x + 2$$

$$5 = x + 2 \quad *$$

$$3 = x$$

\* These are linear equations that needed solving.

b.  $10^{3x+2} = 10,000$

$$\log 10000 = 3x + 2$$

$$4 = 3x + 2 \quad *$$

$$2 = 3x$$

$$2/3 = x$$

c.  $5(10^{3x+2}) + 6 = 506$

$$5(10^{3x+2}) = 500$$

$$10^{3x+2} = 100$$

$$\log 100 = 3x + 2$$

$$2 = 3x + 2 \quad *$$

$$0 = 3x$$

$$0 = x$$